Homogeneity and homotheticity of preferences 2

Discuss the homogeneity and homotheticity of the preferences represented by the following utility functions:

1. $u(q_1, q_2)$ such that $u(tq_1, tq_2) = t^r \cdot u(q_1, q_2)$, with $t > 0$.
2. $u(q_1, q_2)$ such that $u(tq_1, tq_2) = g(f(q_1, q_2))$, with $g'(\cdot) > 0$ and $f(\cdot)$ homogeneous of degree r, $t > 0$.
3. $u(q_1, q_2) = \alpha q_1 + \ln q_2$, where $\alpha > 0$.
4. $u(q_1, q_2) = \alpha - \frac{1}{q_1^\beta q_2}$, with $\alpha, \beta > 0$.

Solutions

The preferences are homothetic if the MRS between any two goods—goods 1 and 2, in the present case—is a function that depends only on the consumption ratio $\frac{q_2}{q_1}$, and conversely, not on the absolute quantities of the goods. Consequently, if it doubles, triples, etc., the amount of both goods, the MRS is not modified.

Preferences are homothetic when: (a) the isoquants are straight lines that start from the origin, and (b) the Marshallian demand functions resulting have an income elasticity equal to 1.

The known preference sets of homothetic preferences are the preferences for substitute goods, the preferences for complementary goods, the Cobb-Douglas type preferences, the CES preferences, etc. On the contrary, the quasilinear preferences are not homothetic.

1. **In this case, we are dealing with homogeneous preferences (of degree r), and homothetic preferences.** To see that they are indeed homothetic preferences, it is enough to take into account that the slope of the indifference curves resulting from a utility function that is homogeneous of degree 0 in q_1 and q_2, $\text{MRS}(q_1, q_2) = -\frac{u_1}{u_2} = -t^0 \left(-\frac{u_1}{u_2}\right)$.

2. In this case, it is immediate to verify that $\text{MRS}(q_1, q_2) = -\frac{u_1}{u_2} = \frac{g'}{g'} \frac{\dfrac{\partial f}{\partial u_1}}{\dfrac{\partial f}{\partial u_2}} = -\frac{g'}{g'} \frac{t^r u_1}{t^r u_2}$, that is, the slope of an indifference curve from the utility function u is equal to the slope of an indifference curve from the function $g(\cdot)$. **It is, in essence, homothetic preferences, although not necessarily homogeneous.**

3. **These are quasi-linear preferences, and therefore, they are not homogeneous; not even homothetic.** That they are not homogeneous is derived from the fact that $u(tq_1, tq_2) \neq t^r \cdot u(q_1, q_2)$. And that they are not homothetic can be proved by seeing that $\text{MRS}(q_1, Fq_2) = -\alpha q_2$ is not a homogeneous function of degree 0 in q_1 and q_2.

4. **u is not a homogeneous utility function, although it is homothetic**, because $\text{MRS}(q_1, q_2) = -\frac{u_1}{u_2} = -\alpha q_2$ is a homogeneous function of degree 0 in q_1 and q_2. The preferences represented by such a function are, therefore, homothetic.